Polynomial Forecasting of Securities' Prices

Overview

In this study we develop a methodology for evaluating forecasting methods applicable to trading securities. The examples cited are all forecasts of future prices made by extrapolating a polynomial curve that has been fitted to historical data from the security in question. The methods here should be extensible to other forecasting techniques.

As discussed in the prior Summa Management study "Plots Specification for Cause and Effect Analysis for Securities Price Movements", to be valid, forecasting techniques must be evaluated in the context with which they will be used. This includes how the forecast will be interpreted and how the trading system utilizing it works.

Incorporating these concepts, this methodology entails the following steps when applied to polynomial forecasting:

1. Financial performance is measured while back-testing over a period which includes a minimum of one full economic cycle
2. Use regression analysis to fit a polynomial to split-adjusted prices for the target security for the last N trading days prior to the test day[1]
3. Plug the test day and the next eligible trading day into the daily fitted curve formula and determine if the projection is for the price to rise or fall between the two
4. If the price is projected to rise, the study's funds are invested in a long position for the security; if projected to fall, the study's funds are invested in a short position[2,3]
5. Each flip of position (to go from buy long to sell short, or the reverse) is assumed to cost 0.25%[4]
6. The effective investment yield (Compound Annual Growth Rate, CAGR) is computed for the previous M months and plotted with the security price history, and the CAGR is computed for the full term of the study as a single figure-of-merit
7. Parameters of the study are swept across a range to find the conditions which result in the peak return (CAGR)
8. At the peak return conditions, graph how the accuracy of the forecast fared short versus longer term and on rising versus falling markets, particularly for big market swings (where it likely that the most money can be made if the projection is correct)

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[1] During back-testing, the test day is sequentially swept from the start to the finish of the back-testing interval.
[2] Note that many securities have an inverse Exchange Traded Fund (ETF) that shorts the targeted security. For the S&P 500, one such fund is ticker "SH".
[3] If the prediction is correct, regardless of whether it is up or down, money will be made (neglecting spread/commissions).
[4] This encompasses the buy/sell spread plus the commission. For this to be realistic, the securities being traded must be sufficiently liquid, their trading volumes sufficiently high relative to the trades of this study, and the size of each trade sufficiently large to amortize fixed fees per trade.

The earlier study cited above showed graphically examples of polynomial forecasting of various orders (including order zero, which is a straight line), in its discussion on Curve Fitting. Additionally, it showed the effect of using different intervals (number of trading days prior to the forecast) to train the fitting function.

The analysis runs over a 5 year period (2006 through 2010) and varies parameters across 4 different dimensions, as follows:

1. Order [1 (= straight line) to 6 (= 5-humped curve)] of predictive polynomial
2. Training time (6 to 20 trading days)
3. Trading cycle (once per trading day to once every 6 trading days)
4. Type of security (broad stock index, blue-chip, volatile small-cap stock)

Study Summary

Security (all stock-based)	Buy and hold 5 year annualized return	Best 5 year forecasting annualized return	Poly-nomial order	Training interval (trading days)	Trade interval (trading days)
Index (S&P 500)	-0.3%	+12.3%	1	25	3
Index (DJI)	+1.0%	+12.2%	2	50	7
Index (NASDAQ composite)	+2.4%	+14.3%	6	45	7
Blue chip (IBM)	+12.0%	+18.6%	6	40	7
Volatile small-cap (Vitesse)	-33.6%	+131.9%	2	10	2

Analysis

The table above shows that the right choice of polynomial prediction parameters consistently beat the market, but that the optimal order of the polynomial, the training interval, and the trade interval were not consistent across the securities tested.

Figures 1 through 5 are more detailed summaries of the back-testing on the various securities. The table on the top of these figures shows how the CAGR varies as a function of the order of forecasting polynomial and the amount of history for which it is optimized (i.e. the training period). What is evident, particularly when this table is plotted in the adjacent 3-D view, is that the CAGR does not behave in a nice smooth fashion, monotonically rising to a peak in the polynomial-order/training-day space. This implies that the solution to obtain peak CAGR is fickle, and not robust. A minor deviation in either dimension, but especially the polynomial order, results in a major change in the CAGR. In the case of the S&P 500, moving 1 notch up in polynomial order and two notches down in training period moves one from the maximum CAGR to the minimum CAGR. Also note that multiple (though lesser) CAGR peaks exist.

The trade frequency table, in the middle of each summary, shows a strong dependency on how frequently one trades. Look particularly at the DJI, where changing from trading every 6 days to trading every 7 days moved the annual return from near zero to over 12%. Also note that these algorithms predicted the correct market direction just barely better than a flip of a coin (up to 56% correct), and many times they were worse (down to 41% correct). This also shows that this prediction method is not systemically robust, and what we are seeing is noise and coincidence.

The chart at the bottom of each of these summaries shows what points in time this trading system made money, and when it lost money; all in comparison to what the price of the underlying security was doing. For the S&P 500, there were two regions where the system consistently made money (as evidenced by the 20 day CAGR plots all being above 0% (out of the red-shaded region). These are July 2006 through February 2007, and October 2008 through December 2008, with one gap. These regions are magnified in Figures 16 and 17. The S&P was slowly but consistently trending up in the first, and falling rapidly in the second, except for an uptick that corresponded to the previously mentioned gap in positive returns (because the trading system was holding a short position).

The next set of graphs, Figures 6 & 7 for the S&P, and corresponding pairs for the other securities evaluated, show how the prediction algorithm performed versus how far out in the "future" the prediction was made, and how well the prediction algorithm performed in getting the market movement direction correct for various slopes of market movement (% change at the next eligible trading day). As would be expected, most market movements between trading days were small (between -½% and +½%), while a few were ±5% or greater. The character of the curves did not change significantly as one looked forward from 1 to 6 days out, although a higher percentage of the changes were greater (no longer in the -½% to +½% bin) the further out one went.

What is curious is that down-market movements were predicted by these polynomials much more accurately than up-market movements. This is shown clearly on the bottom chart in Figure 7, which looks at how the predictions worked on big (5% or more per trading interval) changes. I attribute this to the character of the market over this interval. When it fell, it fell fast and with conviction; when it rose, it rose slowly and more uncertainly.

Areas for Further Work

Potential other forecasting methods that can be evaluated for their performance within a trading system are enumerated on the website http://www.ipredict.it/ForecastingMethods.aspx and listed below. These all extrapolate future datapoints algorithmically from historical datapoints, as opposed to using external data, such as news items, to form the basis of the forecast.

It should be useful to determine if any of these can consistently produce good trading returns, and understand the conditions under which promising algorithms perform well and where they don't. This additional insight should allow for even better returns.

Classical Algorithms
1. Simple Moving Average
2. Geometric Moving Average
3. Triangular Moving Average
4. Parabolic Moving Average
5. Double Moving Average
6. Exponential Moving Average
7. Double Exponential Moving Average
8. Holt's Double Exponential
9. Triple Exponential Moving Average
10. Holt's Triple Exponential
11. Adaptive Response Rate Exponential Smoothing
12. Holt Winter's Additive
13. Holt Winter's Multiplicative
14. Holt Winter's Modified Multiple Seasonalities
15. Additive Decomposition
16. Multiplicative Decomposition
17. Sparse Series Croston's Exponential
18. Linear Trend / Regression

Curve and Bayesian Model Fitting
19. Linear Trend And Additive Seasonality
20. Linear Trend And Multiplicative Seasonality
21. Linear Trend And Multiple Seasonalities
22. Polynomial
23. Logarithmic
24. Exponential

Wavelet Smoothing and Forecasting
25. Frequency Identification
26. Haar De-noising
27. Daubechies Linear De-noising
28. Daubechies Exponential De-noising
29. Wavelet Forecasting

Additional Functions and Algorithms
30. Random Number Generation
31. Wavelet Transforms
32. Fourier Transforms
33. Hurst Exponent

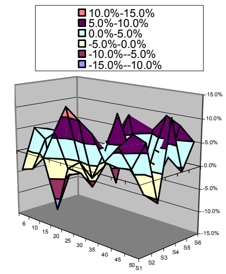
Advanced Algorithms
34. Fractal Projection
35. Active Moving Average

S&P 500

Trading days prior to prediction date used to formulate curve fit

Order of Polynomial	6	10	15	20	25	30	35	40	45	50
1	-4.0%	-3.2%	0.1%	6.5%	12.3%	10.1%	8.0%	2.7%	6.5%	9.5%
2	5.0%	2.9%	-12.6%	-2.3%	-0.2%	-2.9%	3.4%	0.8%	1.2%	7.3%
3	4.2%	4.2%	-5.4%	-4.2%	-6.5%	-1.1%	10.9%	3.1%	-4.9%	-5.8%
4	2.9%	3.2%	-1.5%	-0.3%	1.6%	3.6%	-1.6%	-5.3%	-2.8%	11.1%
5	1.6%	3.0%	-5.3%	0.2%	4.1%	9.0%	-3.7%	2.6%	1.6%	-0.8%
6	1.9%	-3.8%	-1.5%	-8.8%	3.3%	5.4%	5.7%	9.4%	7.8%	4.9%

Max CAGR (Compound Annual Growth Rate) over full dataset (5.00 years).



Trading days for estimate: 25
Order of polynomial (1 to 6): 1
Specific case shown in detail below and on "1-week Prediction" sheet

Trade frequency (every N trading days, where this is N)	1	2	3	4	5	6	7	Once (buy and hold)
Percent of time market direction predicted correctly	53%	53%	52%	54%	55%	55%	56%	
CAGR over full 5.00 year trading interval	4.7%	6.3%	12.3%	7.6%	4.9%	5.2%	5.1%	-0.3%

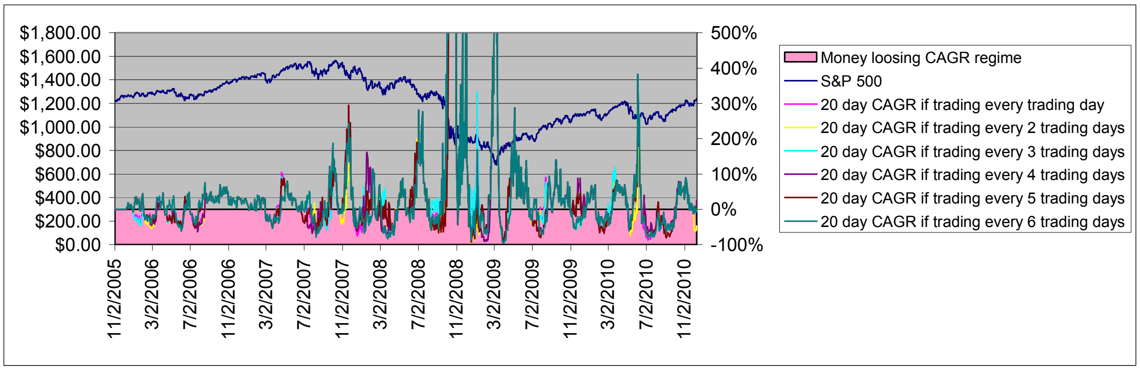


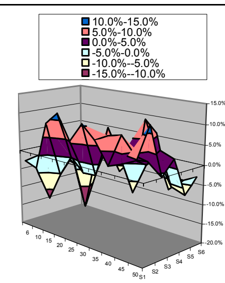
Figure 1

DJI

Trading days prior to prediction date used to formulate curve fit

Order of Polynomial	6	10	15	20	25	30	35	40	45	50
1	-2.4%	-3.5%	8.8%	11.9%	5.7%	10.5%	7.3%	1.9%	3.1%	4.8%
2	-1.7%	-12.6%	-5.3%	0.1%	-3.1%	-2.9%	7.6%	8.5%	4.3%	12.2%
3	-3.4%	-0.5%	-1.1%	-4.0%	-2.7%	8.3%	1.3%	1.8%	2.2%	-0.8%
4	-3.7%	-4.6%	-15.9%	-0.5%	4.0%	0.0%	-8.3%	-0.3%	4.1%	2.8%
5	-2.5%	-5.1%	-7.0%	1.0%	1.4%	-0.3%	9.8%	0.8%	-5.8%	-7.0%
6	-5.7%	0.0%	1.7%	-7.0%	-3.1%	0.1%	5.6%	-8.4%	-5.1%	-3.4%

Max CAGR (Compound Annual Growth Rate) over full dataset (4.90 years).



Trading days for estimate: 50
Order of polynomial (1 to 6): 2
Specific case shown in detail below and on "1-week Prediction" sheet

Trade frequency (every N trading days, where this is N)	1	2	3	4	5	6	7	Once (buy and hold)
Percent of time market direction predicted correctly	53%	50%	51%	52%	53%	53%	54%	
CAGR over full 4.90 year trading interval	-3.8%	-4.2%	-0.9%	2.9%	2.2%	0.8%	12.2%	1.0%

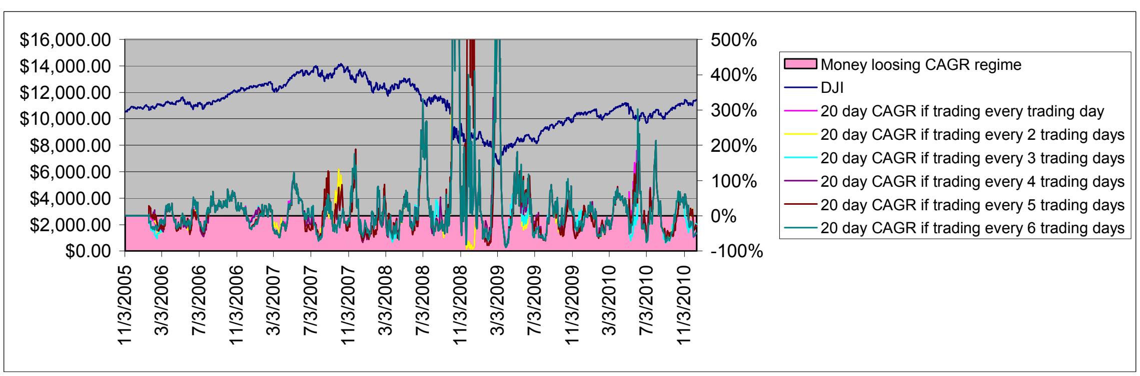


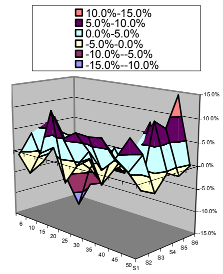
Figure 2

NASDAQ

Trading days prior to prediction date used to formulate curve fit

Order of Polynomial	6	10	15	20	25	30	35	40	45	50
1	-0.6%	-4.2%	1.1%	11.0%	5.6%	7.5%	7.2%	7.6%	4.2%	8.9%
2	3.8%	-2.3%	1.7%	-5.6%	-1.0%	2.9%	-0.5%	2.4%	0.6%	4.9%
3	-1.6%	7.5%	-6.1%	-1.1%	-0.3%	-5.3%	3.6%	-0.8%	-4.0%	-3.7%
4	3.5%	0.5%	-13.3%	-3.8%	0.7%	-2.7%	2.3%	2.4%	9.8%	-0.3%
5	1.5%	-4.6%	-9.2%	-8.1%	-2.4%	-2.0%	-1.3%	-2.0%	7.8%	-1.7%
6	0.8%	-1.9%	-9.1%	-5.7%	-5.7%	1.2%	7.4%	-3.4%	14.3%	-0.3%

Max CAGR (Compound Annual Growth Rate) over full dataset (4.92 years).



Trading days for estimate	45	Specific case shown in detail below and on "1-week Prediction" sheet
Order of polynomial (1 to 6)	6	

	1	2	3	4	5	6	7	Once (buy and hold)
Trade frequency (every N trading days, where this is N)	1	2	3	4	5	6	7	Once (buy and hold)
Percent of time market direction predicted correctly	51%	50%	50%	49%	50%	50%	51%	
CAGR over full 4.92 year trading interval	-17.0%	-17.7%	-9.3%	-15.1%	-7.7%	-4.5%	14.3%	2.4%

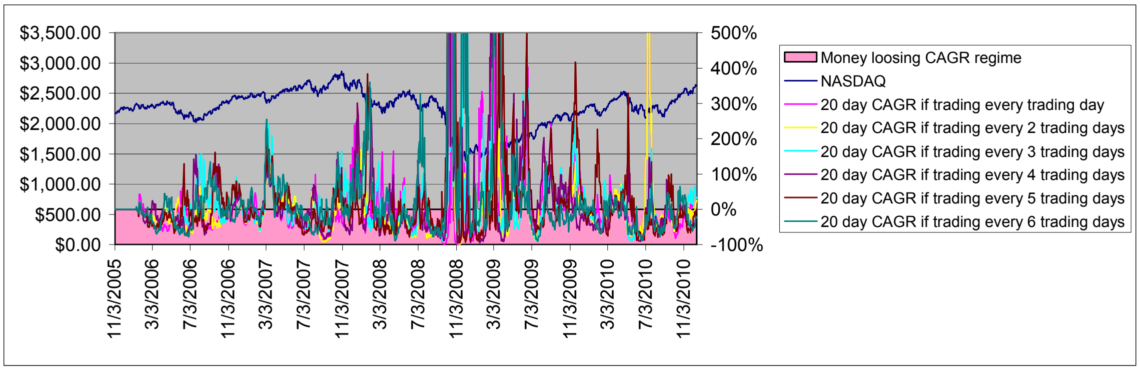


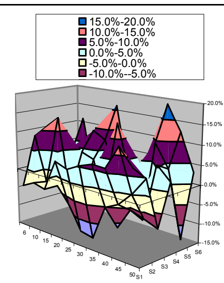
Figure 3

IBM

Trading days prior to prediction date used to formulate curve fit

Order of Polynomial	6	10	15	20	25	30	35	40	45	50
1	-4.1%	-2.6%	-3.3%	4.9%	3.6%	5.3%	6.5%	6.4%	12.3%	8.0%
2	8.6%	-3.0%	-3.3%	-7.5%	-13.5%	-14.2%	-5.1%	-3.4%	0.1%	4.8%
3	10.7%	13.4%	-7.1%	-12.9%	1.0%	-2.1%	-11.5%	-5.8%	-7.0%	-8.4%
4	8.7%	4.4%	4.2%	3.0%	-8.7%	13.8%	0.9%	-9.5%	5.5%	2.2%
5	1.9%	0.6%	9.4%	7.5%	-2.2%	-1.6%	2.5%	9.5%	3.7%	-13.5%
6	10.9%	-5.8%	7.2%	16.1%	-1.1%	-9.8%	1.7%	18.6%	12.3%	2.8%

Max CAGR (Compound Annual Growth Rate) over full dataset (4.94 years).



Trading days for estimate	40	Specific case shown in detail below and on "1-week Prediction" sheet
Order of polynomial (1 to 6)	6	

	1	2	3	4	5	6	7	Once (buy and hold)
Trade frequency (every N trading days, where this is N)	1	2	3	4	5	6	7	Once (buy and hold)
Percent of time market direction predicted correctly	49%	48%	47%	48%	49%	49%	50%	
CAGR over full 4.94 year trading interval	-14.4%	-19.3%	-18.7%	-11.6%	-8.9%	-16.1%	18.6%	12.0%

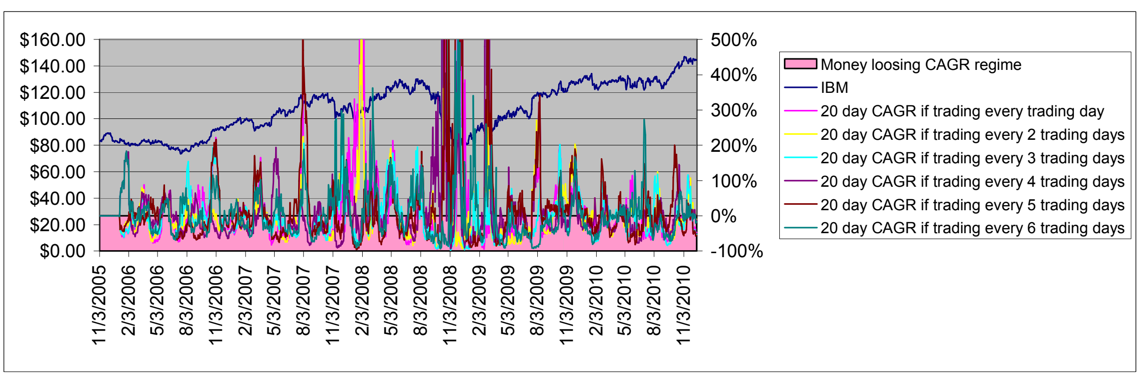
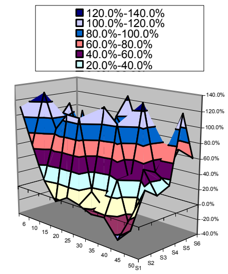


Figure 4

Vitesse



Trading days prior to prediction date used to formulate curve fit

Order of Polynomial	6	10	15	20	25	30	35	40	45	50
1	112.9%	64.8%	21.3%	-3.5%	6.7%	2.1%	-4.2%	-2.6%	-25.5%	11.2%
2	68.3%	131.9%	86.4%	108.6%	107.1%	15.1%	-2.3%	40.6%	-19.7%	36.9%
3	45.6%	89.6%	95.1%	59.4%	101.4%	121.5%	102.9%	29.3%	53.5%	25.1%
4	81.8%	112.1%	59.8%	69.9%	58.5%	54.5%	45.0%	129.9%	45.3%	32.6%
5	4.7%	62.3%	45.7%	44.8%	99.8%	75.4%	102.9%	80.5%	68.4%	119.6%
6	92.6%	36.3%	19.6%	65.5%	124.7%	77.3%	45.6%	26.1%	66.3%	59.1%

Max CAGR (Compound Annual Growth Rate) over full dataset (5.07 years).

Trading days for estimate	10	Specific case shown in detail below and on
Order of polynomial (1 to 6)	2	"1-week Prediction" sheet

Trade frequency (every N trading days, where this is N)	1	2	3	4	5	6	7	Once (buy and hold)
Percent of time market direction predicted correctly	41%	43%	46%	47%	46%	45%	45%	
CAGR over full 5.07 year trading interval	126.7%	131.9%	109.4%	81.8%	12.9%	95.3%	-33.6%	-33.6%

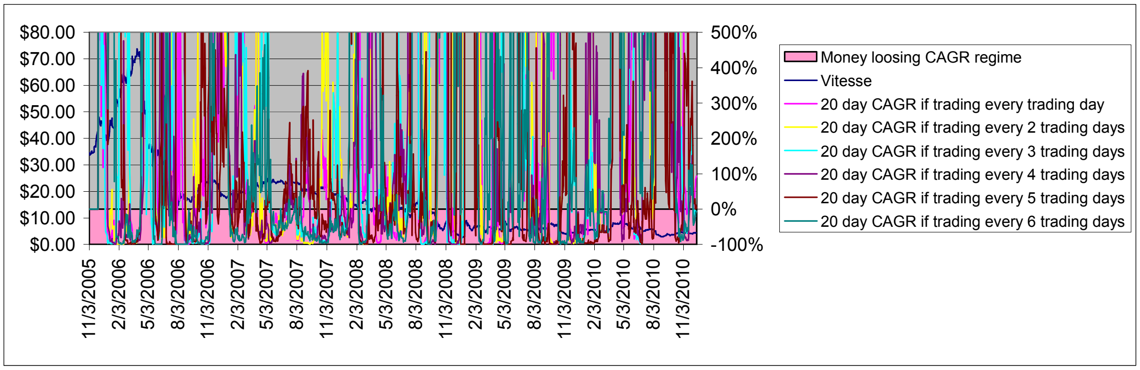
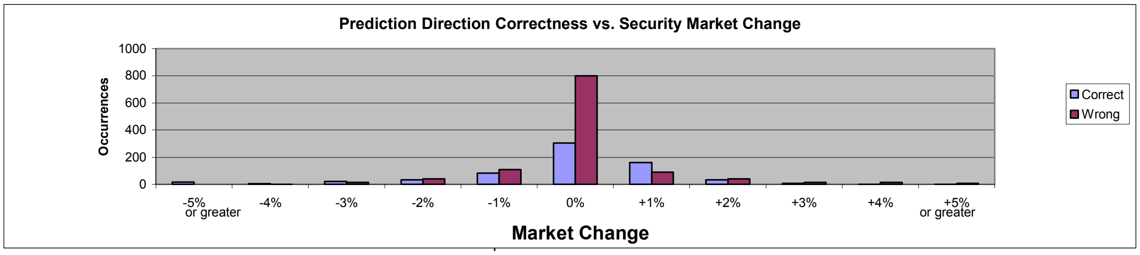


Figure 5

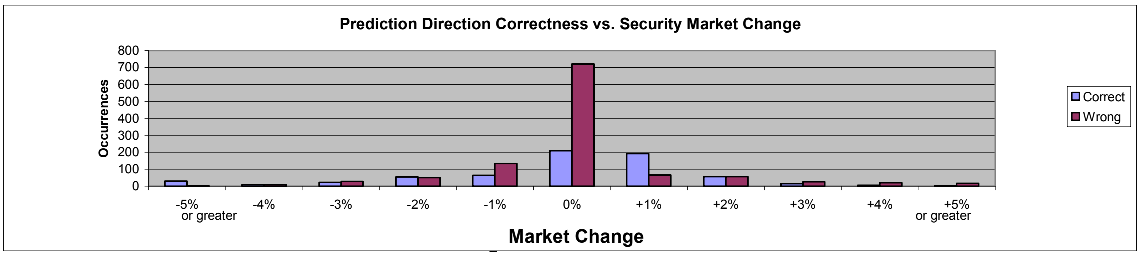
S&P 500

Trading days for estimate: 25 Order of polynomial: 1



1 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	17	6	21	34	83	304	160	34	7	1	1
Wrong	0	1	15	41	109	800	89	40	16	15	9

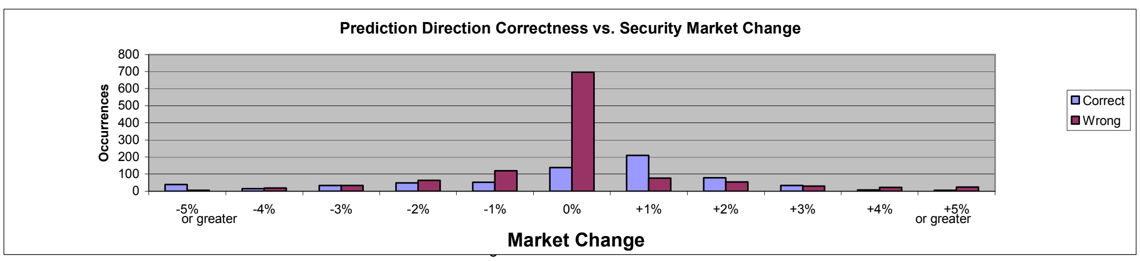


2 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	30	9	24	54	64	211	194	58	15	7	3
Wrong	2	9	28	52	133	722	67	58	26	20	17

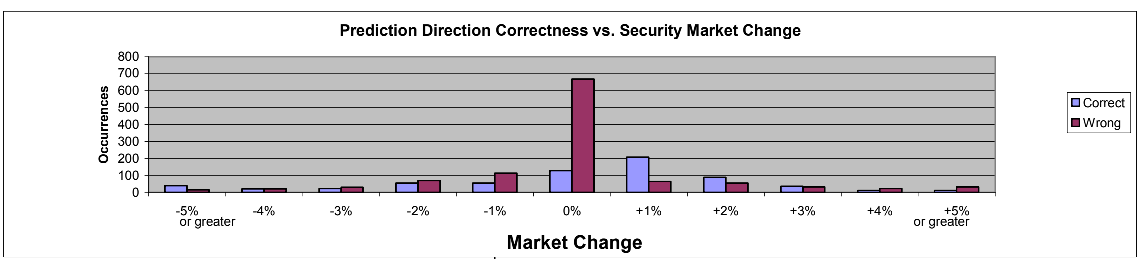
S&P 500

Trading days for estimate: 25 Order of polynomial: 1



3 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	38	15	34	48	53	138	210	77	34	7	6
Wrong	6	18	34	63	120	696	76	54	29	22	25

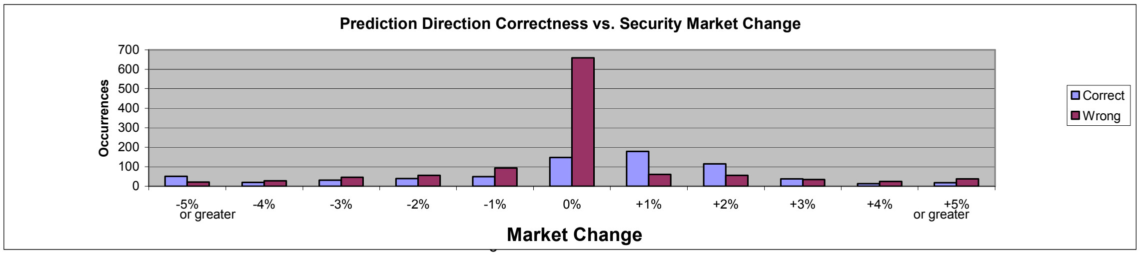


4 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	40	22	24	54	54	129	207	89	36	12	11
Wrong	16	20	30	70	114	669	64	55	32	23	32

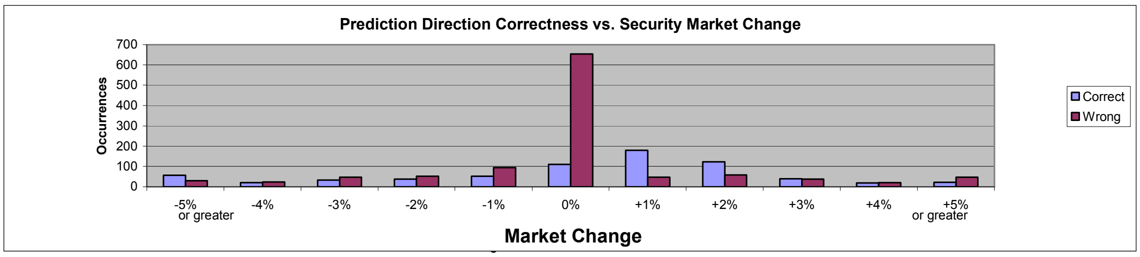
Figure 6

S&P 500 Trading days for estimate: 25 Order of polynomial: 1



5 **Day ahead forecast**

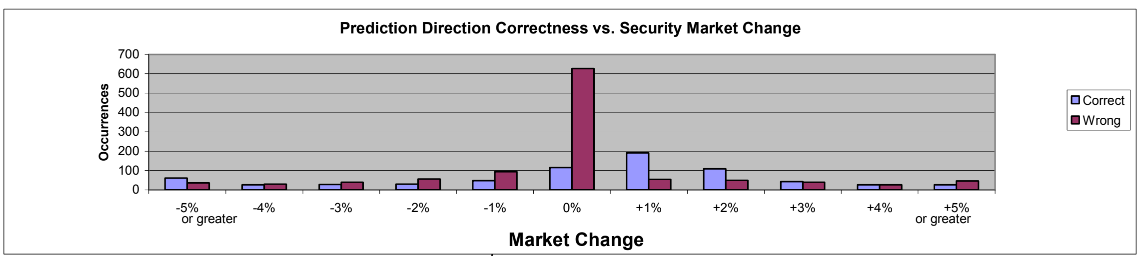
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	50	20	30	38	49	146	178	114	37	13	18
Wrong	21	28	45	56	92	658	61	55	33	24	37



6 **Day ahead forecast**

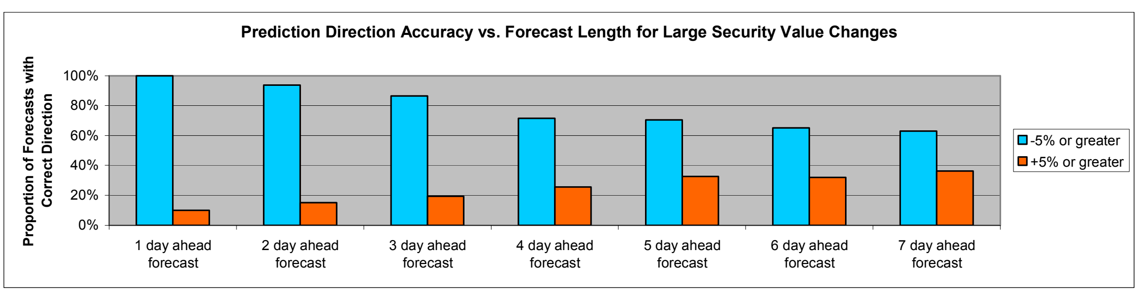
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	56	20	33	37	52	111	180	122	40	18	22
Wrong	30	23	47	52	95	655	47	59	37	20	47

S&P 500 Trading days for estimate: 25 Order of polynomial: 1



7 **Day ahead forecast**

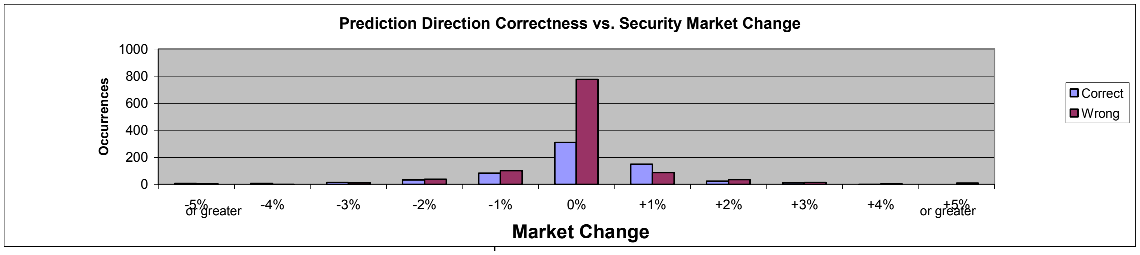
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	61	27	28	30	48	116	191	109	43	27	26
Wrong	36	29	39	55	94	627	54	50	40	27	46



	1 day ahead forecast	2 day ahead forecast	3 day ahead forecast	4 day ahead forecast	5 day ahead forecast	6 day ahead forecast	7 day ahead forecast
-5% or greater	100%	94%	86%	71%	70%	65%	63%
+5% or greater	10%	15%	19%	26%	33%	32%	36%

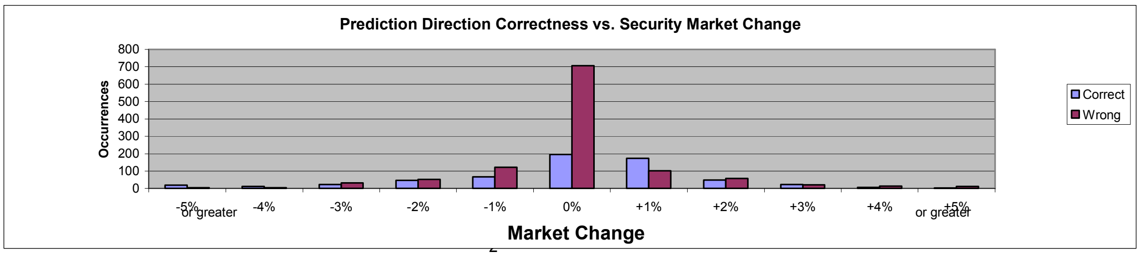
Figure 7

DJI Trading days for estimate: 50 Order of polynomial: 2



1 Day ahead forecast

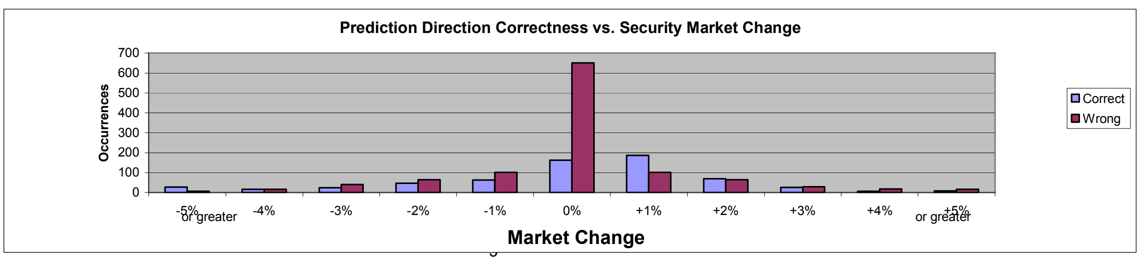
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	9	9	14	34	83	310	151	24	13	2	0
Wrong	3	2	13	39	102	777	89	37	15	4	10



2 Day ahead forecast

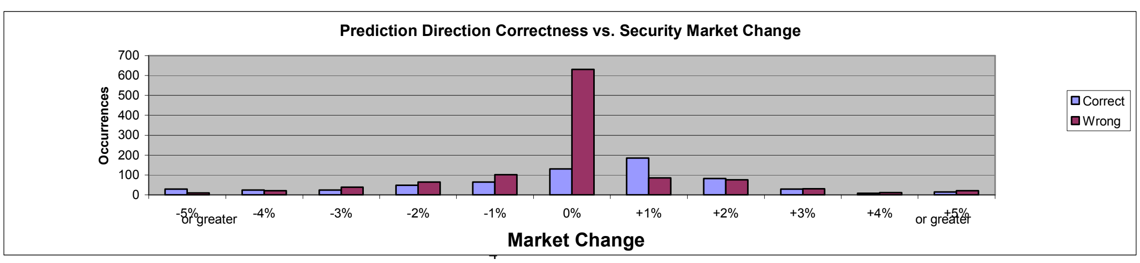
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	20	12	22	46	67	196	173	49	22	5	3
Wrong	4	4	32	52	122	706	101	58	21	14	11

DJI Trading days for estimate: 50 Order of polynomial: 2



3 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	27	16	24	46	63	163	187	68	25	7	9
Wrong	6	15	40	65	101	651	101	65	29	17	15

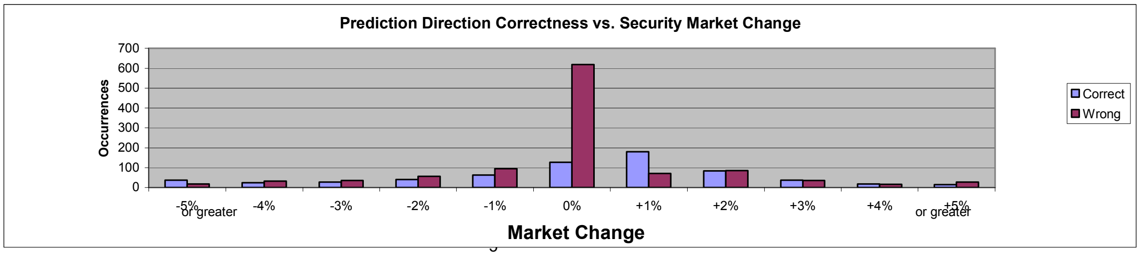


4 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	29	24	24	48	65	131	186	83	29	9	15
Wrong	11	21	39	65	101	632	86	76	31	13	22

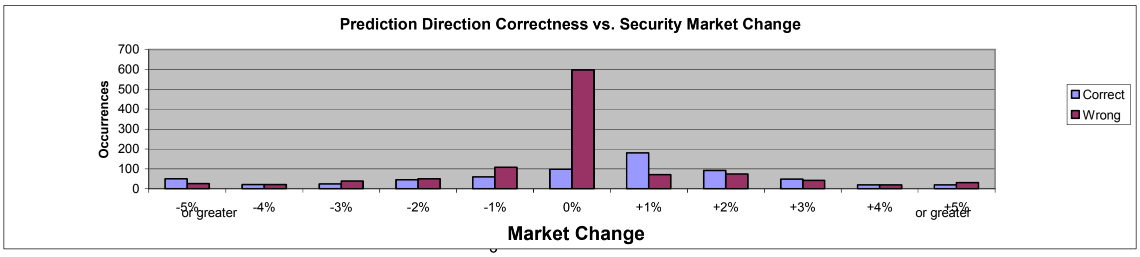
Figure 8

DJI Trading days for estimate: 50 Order of polynomial: 2



5 **Day ahead forecast**

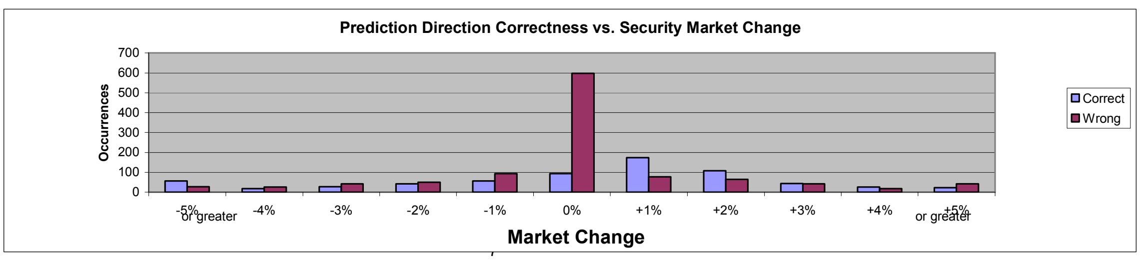
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	38	24	28	40	63	126	179	84	37	18	14
Wrong	17	32	36	56	95	619	70	86	36	15	27



6 **Day ahead forecast**

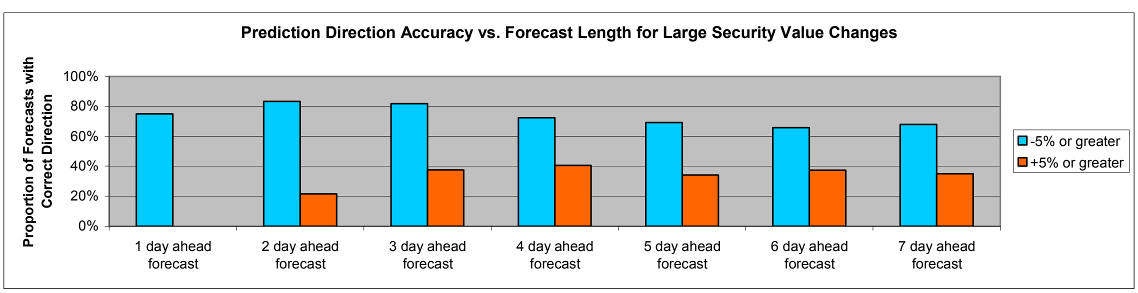
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	50	21	25	45	60	98	181	93	48	19	19
Wrong	26	22	39	50	109	597	71	74	42	19	32

DJI Trading days for estimate: 50 Order of polynomial: 2



7 **Day ahead forecast**

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	57	18	27	41	56	94	174	107	43	25	22
Wrong	27	26	42	50	93	598	77	64	41	17	41

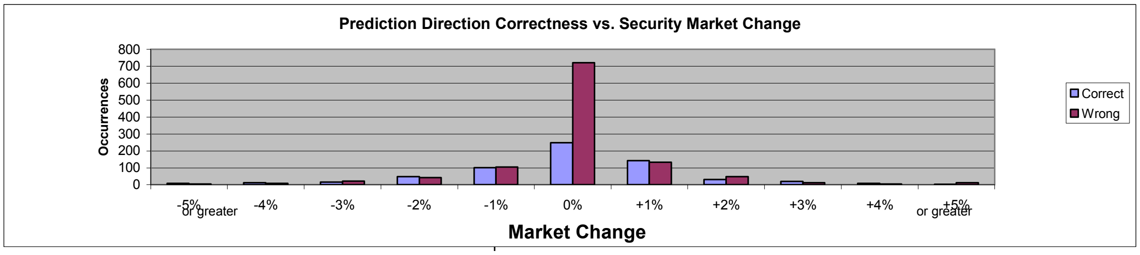


	1 day ahead forecast	2 day ahead forecast	3 day ahead forecast	4 day ahead forecast	5 day ahead forecast	6 day ahead forecast	7 day ahead forecast
-5% or greater	75%	83%	82%	73%	69%	66%	68%
+5% or greater	0%	21%	38%	41%	34%	37%	35%

Figure 9

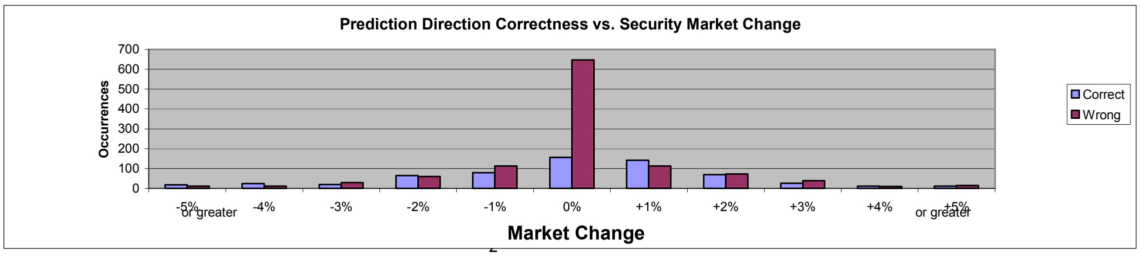
NASDAQ

Trading days for estimate: 45 Order of polynomial: 6



1 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	8	11	16	48	101	249	143	32	19	9	3
Wrong	5	9	21	42	106	721	133	48	13	5	11

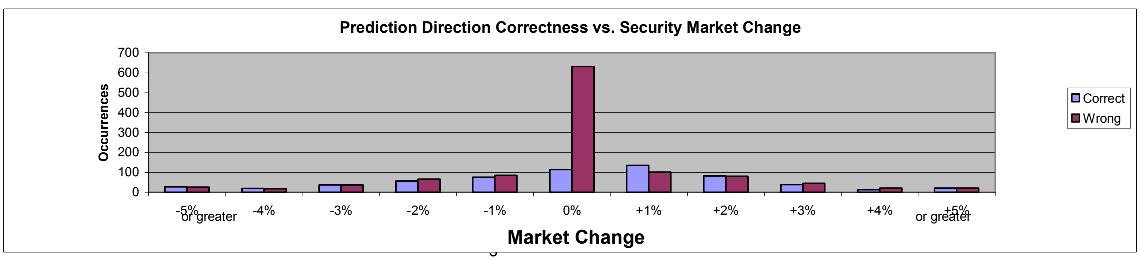


2 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	18	24	20	66	79	156	142	69	26	13	13
Wrong	13	12	30	61	113	647	114	72	39	11	15

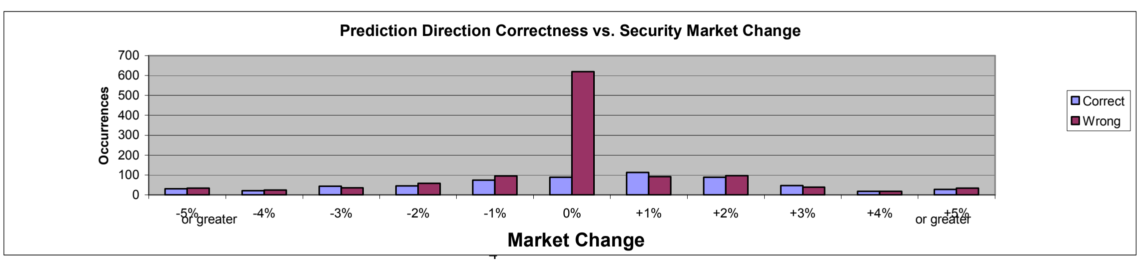
NASDAQ

Trading days for estimate: 45 Order of polynomial: 6



3 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	27	19	37	57	75	115	135	82	39	13	20
Wrong	26	18	37	67	86	632	101	81	45	21	20

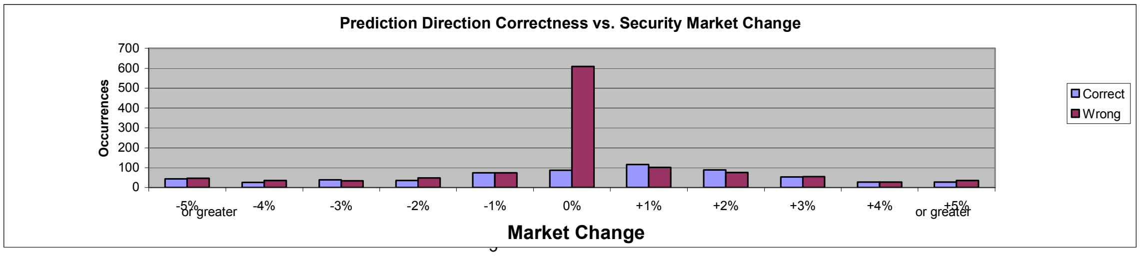


4 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	32	22	43	46	75	90	114	90	47	18	28
Wrong	35	24	36	58	95	619	92	97	40	18	34

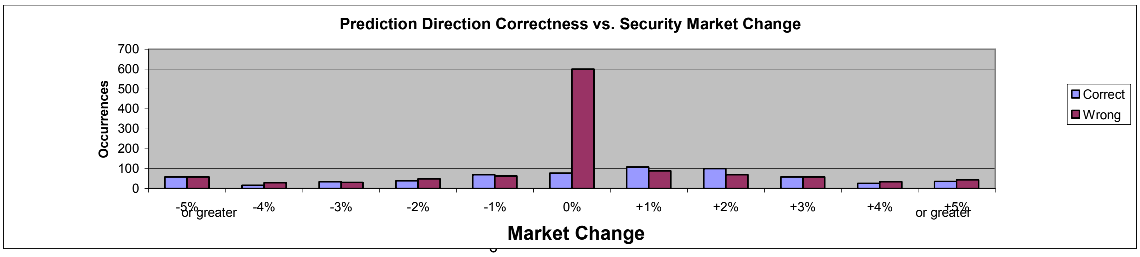
Figure 10

NASDAQ Trading days for estimate: 45 Order of polynomial: 6



5 **Day ahead forecast**

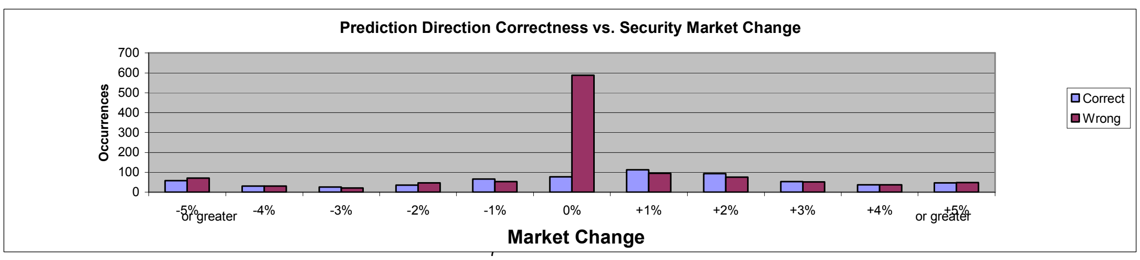
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	43	26	39	35	74	87	116	89	53	27	27
Wrong	46	35	34	48	73	608	101	76	54	27	35



6 **Day ahead forecast**

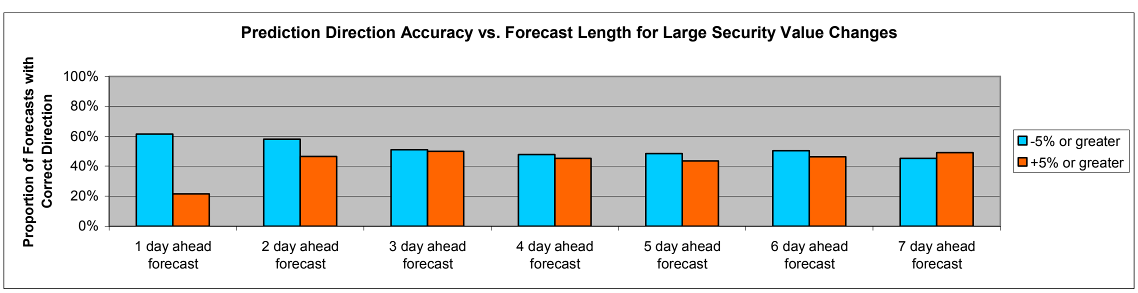
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	59	16	35	39	70	78	108	100	58	26	37
Wrong	58	30	31	49	64	601	89	70	58	34	43

NASDAQ Trading days for estimate: 45 Order of polynomial: 6



7 **Day ahead forecast**

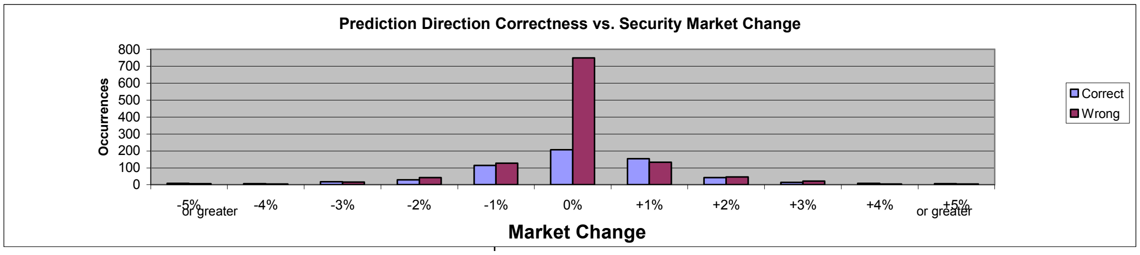
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	58	31	25	36	66	77	113	94	53	38	46
Wrong	70	31	20	46	53	588	95	76	52	37	48



	1 day ahead forecast	2 day ahead forecast	3 day ahead forecast	4 day ahead forecast	5 day ahead forecast	6 day ahead forecast	7 day ahead forecast
-5% or greater	62%	58%	51%	48%	48%	50%	45%
+5% or greater	21%	46%	50%	45%	44%	46%	49%

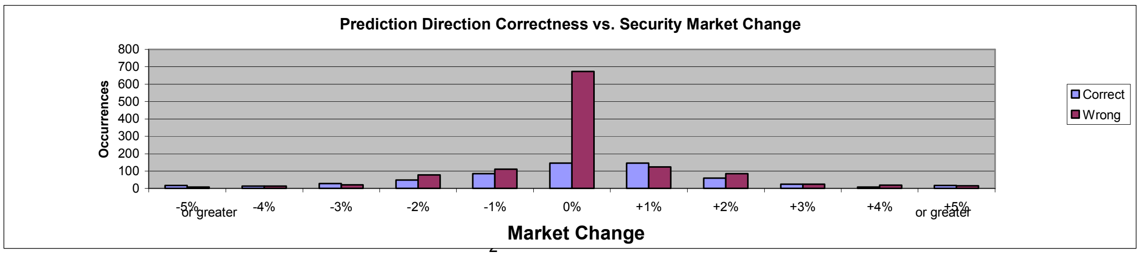
Figure 11

IBM Trading days for estimate: 40 Order of polynomial: 6



1 Day ahead forecast

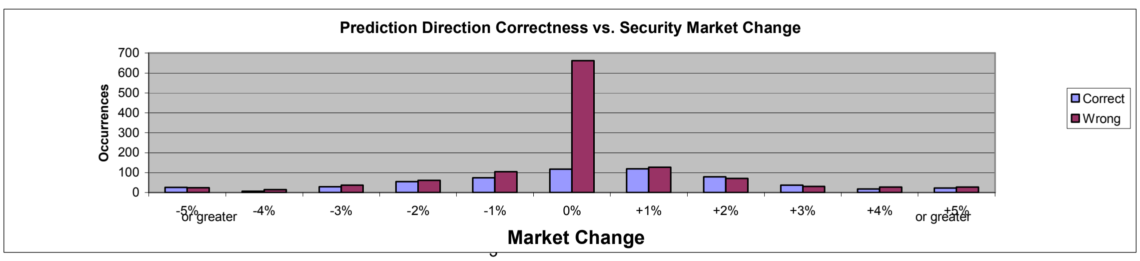
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	9	7	18	28	115	206	153	42	15	9	6
Wrong	6	4	16	43	127	750	134	47	21	4	5



2 Day ahead forecast

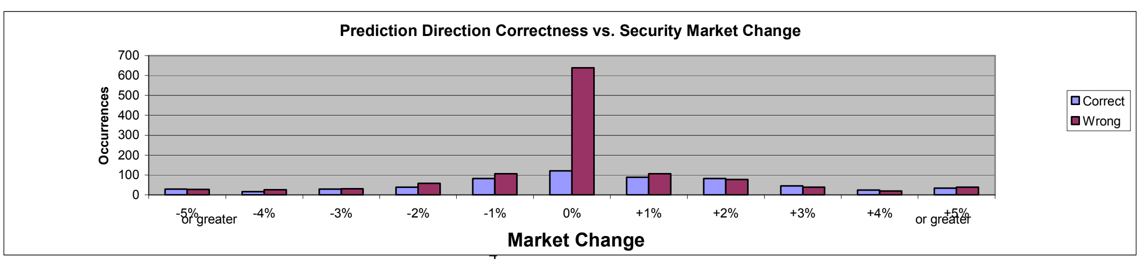
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	16	13	28	49	86	147	147	60	24	9	16
Wrong	9	13	21	77	110	672	123	86	24	20	15

IBM Trading days for estimate: 40 Order of polynomial: 6



3 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	25	7	29	55	73	117	120	78	37	18	22
Wrong	24	14	38	61	105	661	127	70	30	27	27

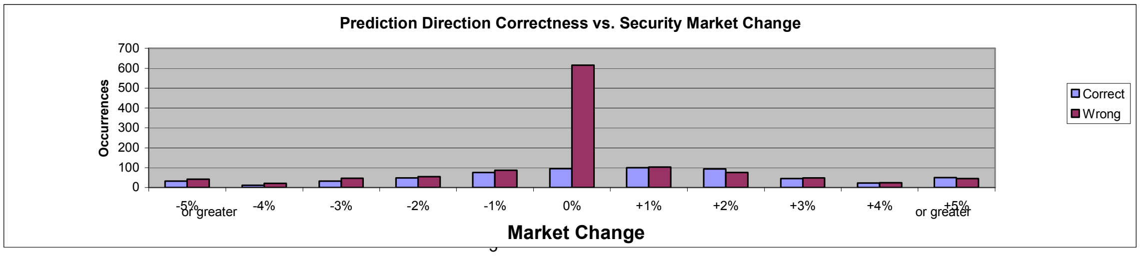


4 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	29	16	29	40	83	121	89	82	45	24	35
Wrong	28	26	32	58	107	639	107	77	39	20	39

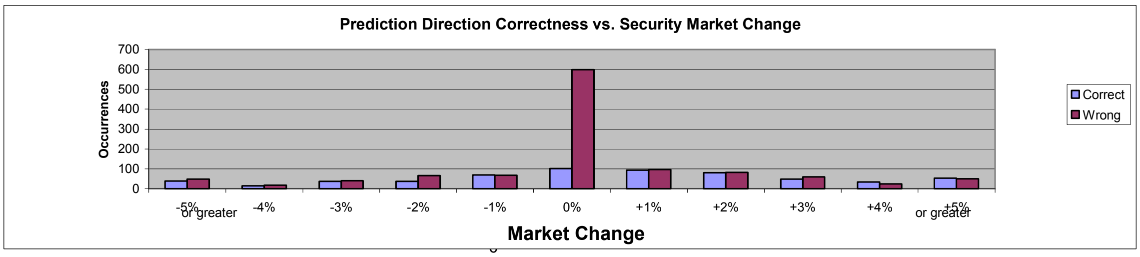
Figure 12

IBM Trading days for estimate: 40 Order of polynomial: 6



5 **Day ahead forecast**

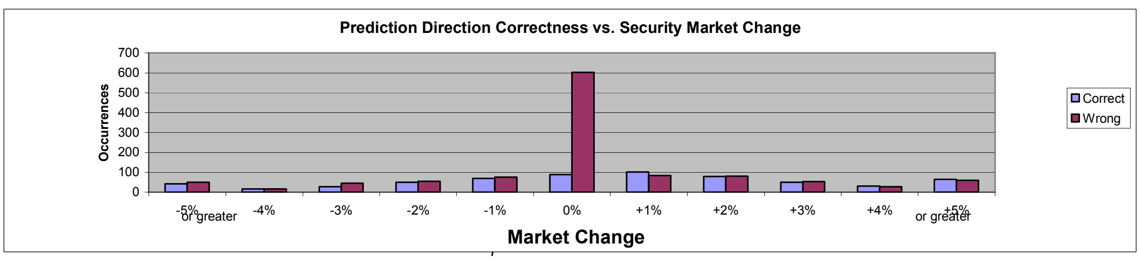
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	32	12	33	48	75	95	100	93	45	23	49
Wrong	41	21	46	54	87	615	103	76	48	24	45



6 **Day ahead forecast**

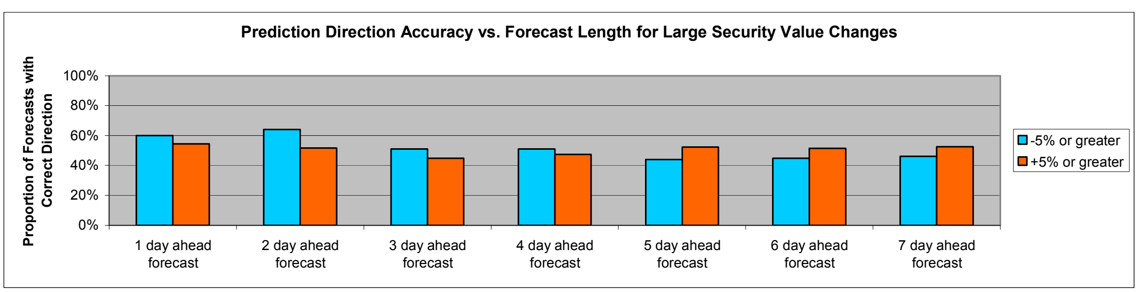
Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	39	14	38	38	70	101	94	81	49	34	53
Wrong	48	18	41	67	68	598	96	82	61	25	50

IBM Trading days for estimate: 40 Order of polynomial: 6



7 **Day ahead forecast**

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	42	16	28	49	69	89	101	79	49	31	65
Wrong	49	16	45	54	75	603	84	81	53	28	59

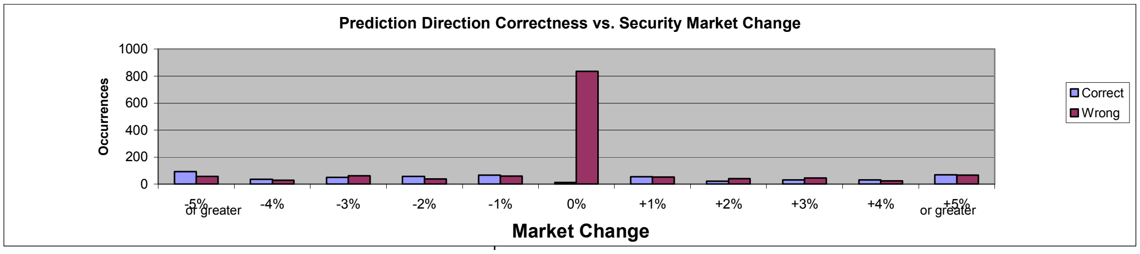


	1 day ahead forecast	2 day ahead forecast	3 day ahead forecast	4 day ahead forecast	5 day ahead forecast	6 day ahead forecast	7 day ahead forecast
-5% or greater	60%	64%	51%	51%	44%	45%	46%
+5% or greater	55%	52%	45%	47%	52%	51%	52%

Figure 13

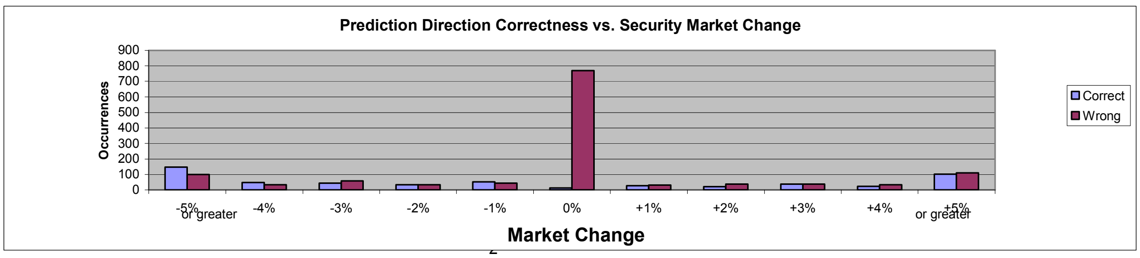
Vitesse

Trading days for estimate: 10 Order of polynomial: 2



1 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	94	35	51	58	67	13	56	22	31	32	70
Wrong	57	28	63	39	59	835	53	41	46	26	66

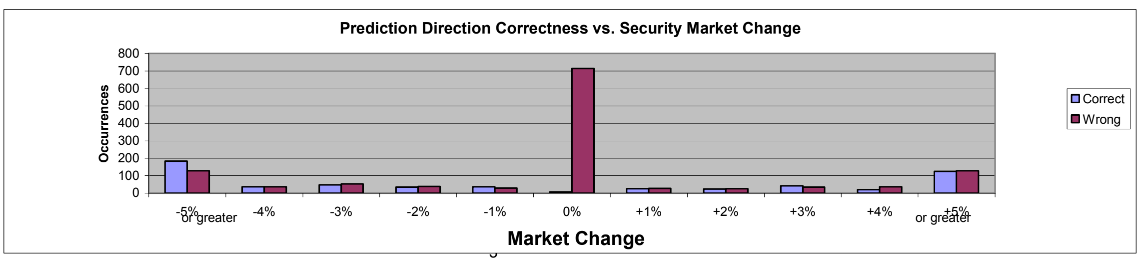


2 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	148	48	45	33	53	12	28	22	37	23	102
Wrong	100	34	60	35	44	768	31	38	38	33	110

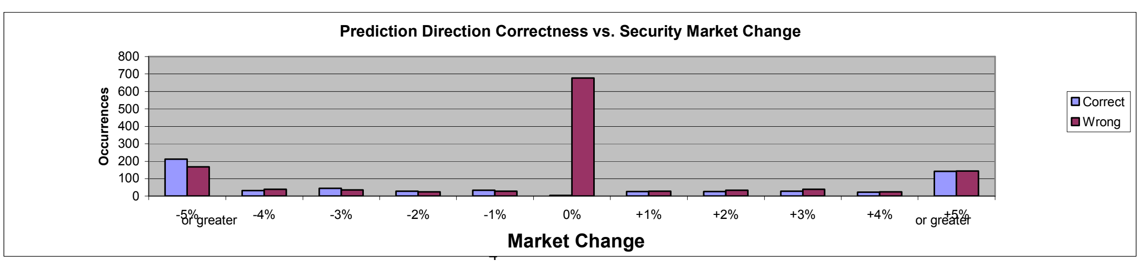
Vitesse

Trading days for estimate: 10 Order of polynomial: 2



3 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	184	36	48	34	38	8	25	23	43	21	126
Wrong	128	37	54	39	30	714	28	26	35	36	129



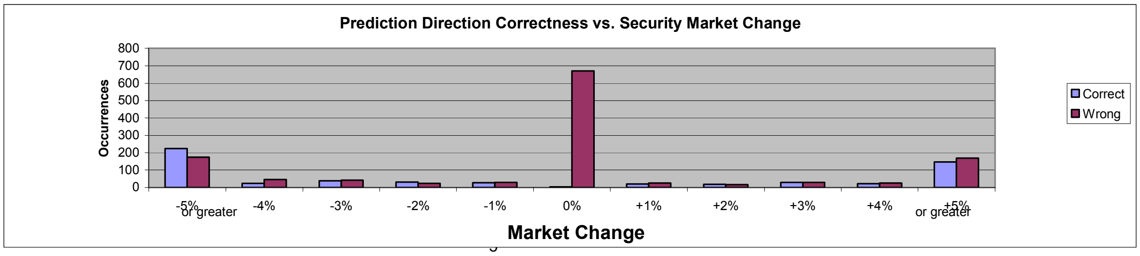
4 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	212	32	44	27	33	4	26	26	28	23	142
Wrong	169	40	36	25	29	677	27	33	40	25	144

Figure 14

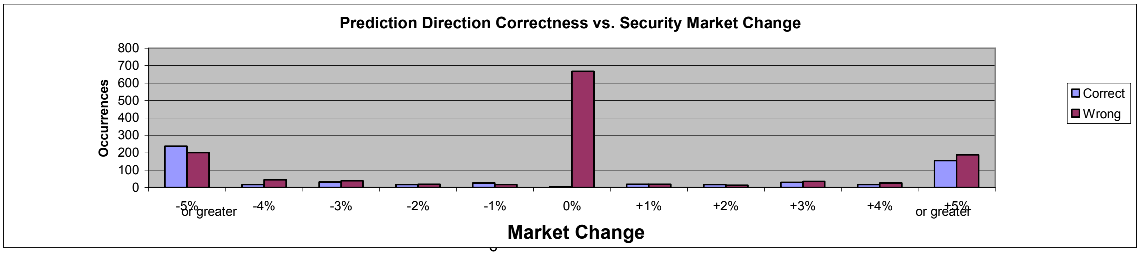
Vitesse

Trading days for estimate: 10 Order of polynomial: 2



5 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	224	24	39	31	28	3	20	19	29	22	147
Wrong	175	46	43	24	29	671	26	17	30	26	169

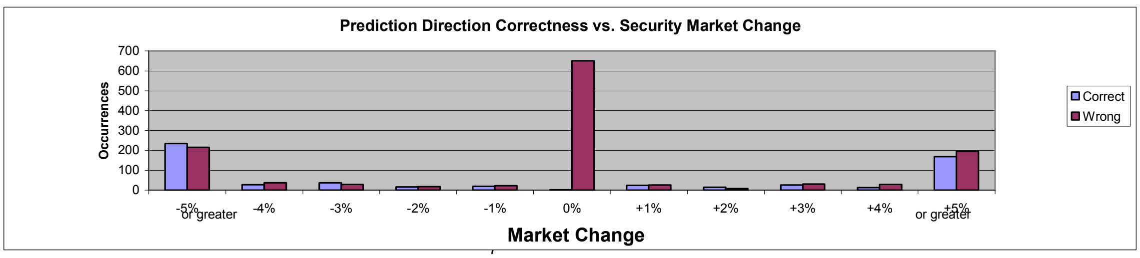


6 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	237	16	32	18	26	4	19	18	30	17	155
Wrong	201	44	39	20	18	667	19	14	35	26	187

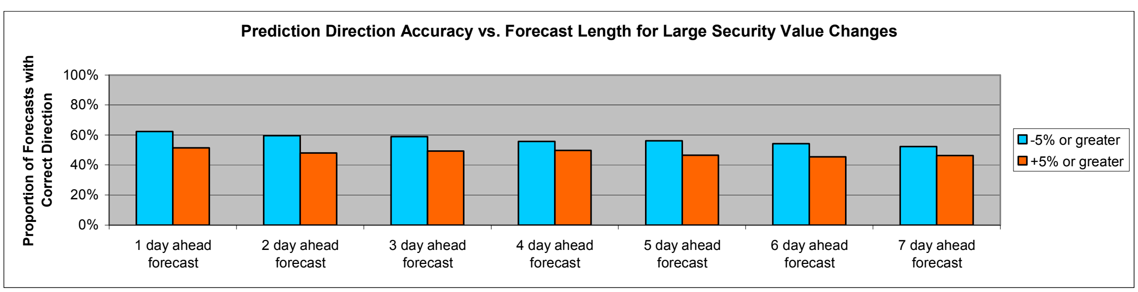
Vitesse

Trading days for estimate: 10 Order of polynomial: 2



7 Day ahead forecast

Market Δ	-5%	-4%	-3%	-2%	-1%	0%	+1%	+2%	+3%	+4%	+5%
Correct	235	28	37	15	19	1	24	14	25	13	169
Wrong	215	38	29	18	22	651	25	8	31	29	196



	1 day ahead forecast	2 day ahead forecast	3 day ahead forecast	4 day ahead forecast	5 day ahead forecast	6 day ahead forecast	7 day ahead forecast
-5% or greater	62%	60%	59%	56%	56%	54%	52%
+5% or greater	51%	48%	49%	50%	47%	45%	46%

Figure 15

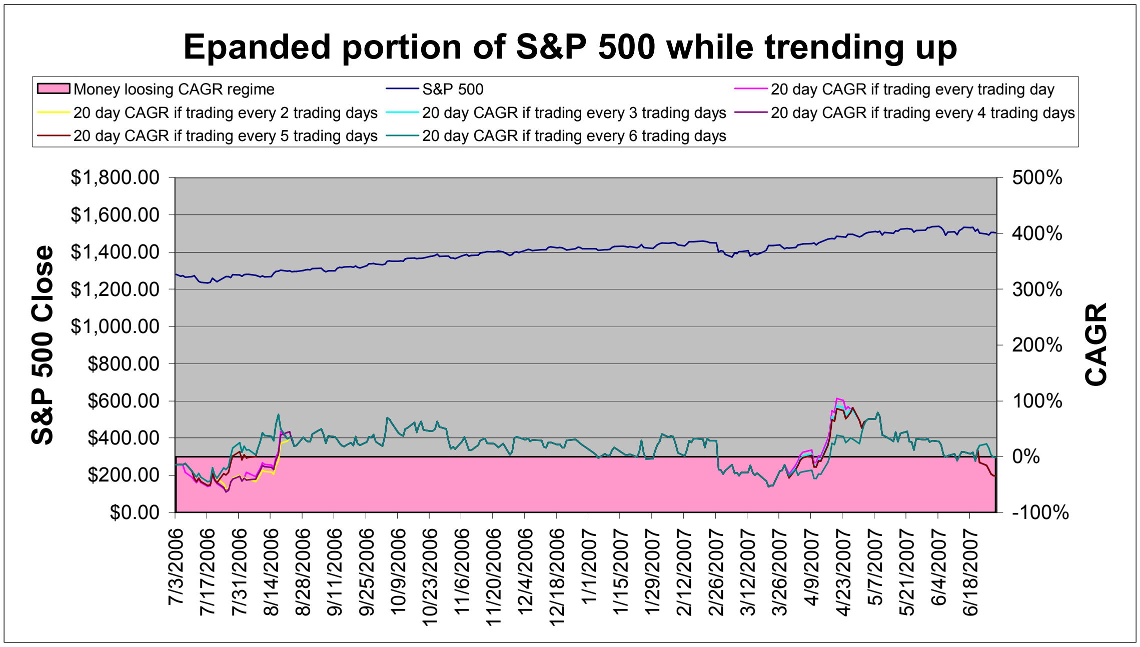


Figure 16

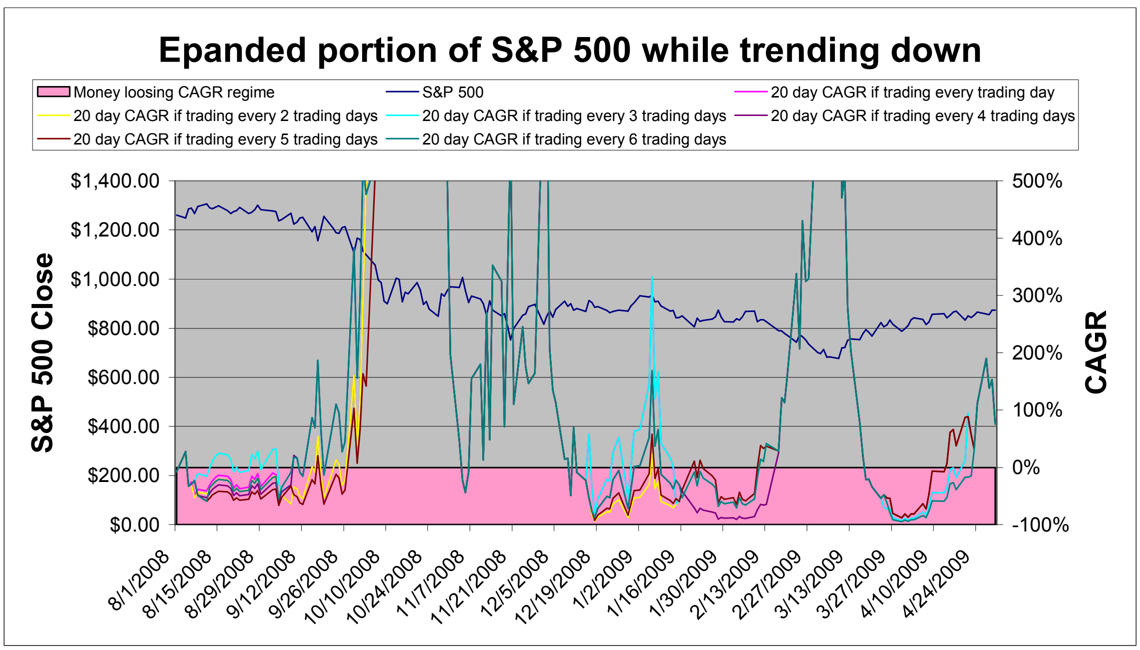


Figure 17

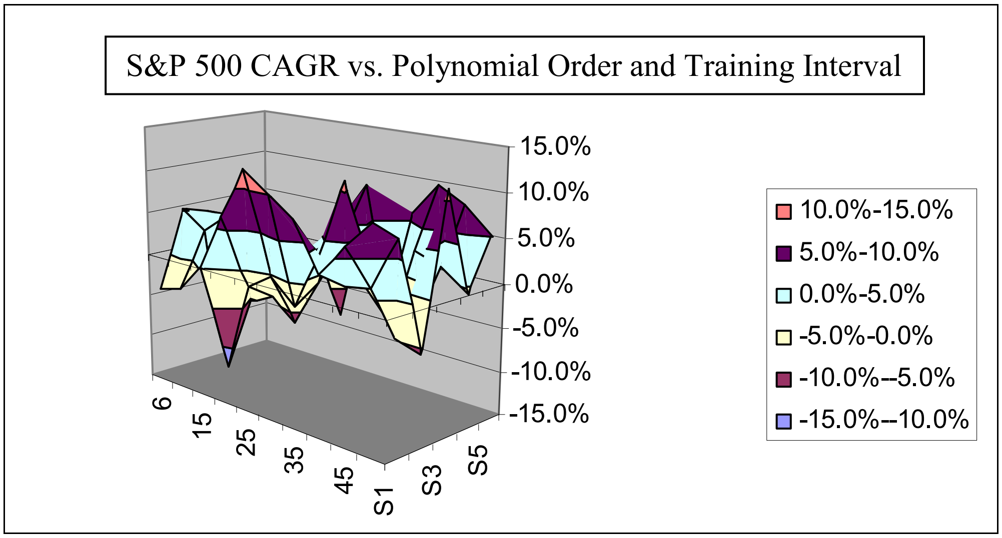

S&P 500 CAGR vs. Polynomial Order and Training Interval
15.0%
10.0%
5.0%
0.0%
-5.0%
-10.0%
-15.0%
6
15
25
35
45
S1
S3
S5
10.0%-15.0%
5.0%-10.0%
0.0%-5.0%
-5.0%-0.0%
-10.0%--5.0%
-15.0%--10.0%

Figure 18